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A NUMERICAL OPTIMIZATION PROCEDURE FOR COMPLEX PIPE AND DUCT NETWORK DESIGN

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ABSTRACT

The design of optimum pipe and duct networks with available procedures is difficult, if not impossible. A more efficient procedure that will automatically produce the optimum design is required. Such a procedure is presented in this article. The design is formulated as a constrained nonlinear optimization problem. This problem is solved using a unique numerical optimization algorithm. The solution entails the calculation of the cross sectional dimensions of the ducts and pipes so that the life cycle cost of the network is minimized. The topology equations are derived using graph theory thereby allowing complex networks with loops to be designed numerically. A duct network consisting of a fan and 35 duct sections is designed according to certain specifications. Using the proposed procedure optimum designs were obtained within seconds on a 33 MHz 486 micro-computer. The procedure was further applied to the optimization of a coal pipeline. It is shown that the optimized solution will cost 14% ($8 million) less than the previous design with conventional design techniques.

KEY WORDS Pipe networks Optimum design Coal pipeline

NOMENCLATURE

E_c	capital cost of network	A_R	reduced incidence matrix
C_i	cost factor	A_T	tree matrix
M_i	mass of pipe or duct section	A_L	link matrix
E_o	operating cost	D_C	cutset matrix
E_d	energy demand rate (cost/kW)	B	loop matrix
E_e	energy rate (cost/kWh)	f_i	friction factor (calculated by Colebrook's equation
$PWEF$	present work escalation factor	ρ	fluid density (kg/m3)
T	total operating hours in a year	L_i	length of duct section (m)
Q_F	fan energy (kW)	d_i	diameter of duct section (m)
I	interest rates	V_i	fluid velocity in duct section (m/s)
D	design life time (years)	δP_i	pressure drop in duct section (Pa)
δE_e	energy rate increase per year	P_F	pressure drop over fan (Pa)
E_L	life cycle cost		
A	incidence matrix		

INTRODUCTION

Complex pipe and duct networks are encountered in most industrial installations and processes, e.g. in power plants, chemical plants, water purification systems, oil refineries and air conditioning systems. Several practical and interrelated considerations have to be taken into account in the

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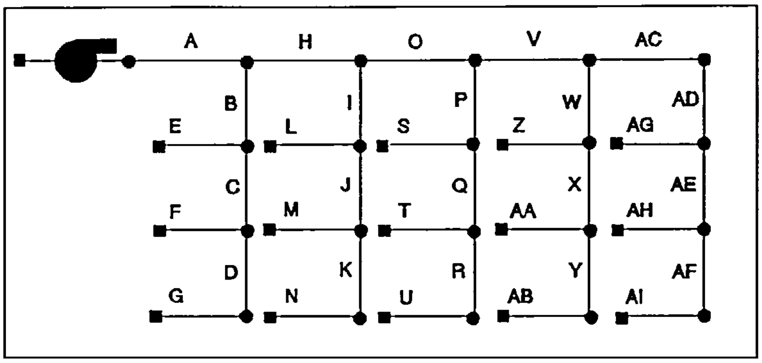

Figure 1 A typical duct network

design of these networks. In order to obtain workable designs, engineers have to continually compromise between practicality and costs. There is thus a need for an easy-to-apply design procedure.

Traditional design methods comprise systematic procedures which engineers follow to obtain a workable design. Although the design specifications are normally satisfied, these methods do not ensure a final best design. The effort necessary to proceed from a workable to the optimum design is often not justified due to the cost of engineering time and the accuracy of empirical data[1]. However, if new optimization codes are as easy to apply as traditional design procedures, the optimization codes will be preferred.

The rapid development of the microcomputer makes the use of numerical optimization techniques for duct network design an attractive proposition. All the design specifications may now be included in one design cycle. The final design will be optimal. This paper proposes such an optimization procedure.

The duct network in *Figure 1* is used to explain the optimization procedure. It consists of a fan and 35 duct sections supplying air to 15 terminals. The problems is to generate an optimal design with respect to duct diameter and fan selection so that the life cycle cost is minimized. The design must also adhere to certain design specifications such as specified flow rates and maximum flow velocities. Usually a workable design for the network is obtained by selecting the duct diameters and calculating the fan pressure necessary to overcome the frictional pressure losses[2]. A fan which can provide the required flow rates at that specific pressure is then selected. If physical constraints are violated or the design is too costly, new diameters are selected. This procedure is repeated until the design specifications and constraints have been satisfied.

This trial and error calculation process for complex networks can become extremely tedious. In order to alleviate the amount of calculations, simulation algorithms have been developed[3-6]. The simulation algorithms have been implemented in commercially available computer programs[7-9]. These programs do not perform design calculations. They are merely tools to facilitate the design process. The engineer specifies the physical layout and dimensions and the program calculates the flow in each branch of the network. Should any constraint or specification be violated, the engineer has to change certain physical parameters and repeat the process.

Much work has recently been done on developing algorithms which include the design variables, e.g. pump speed and valve settings, as additional equations in the simulation[10,11]. In this way it is possible to calculate the design variables that satisfy the specifications. However, these new methods will not necessarily produce the most economical designs. Furthermore, the maximum number of design variables must be equal to the number of specifications. It may, therefore, not be possible to size an entire pipe network using these methods. If the additional design objective of minimum life cycle cost is added, the design process becomes even more complex. One way of solving this problem is through numerical optimization[12–15]. The life cycle cost can be formulated as an objective function which must be minimized. The design specifications are expressed as equality and inequality constraints. This constrained nonlinear optimization problem can then be solved through the use of available algorithms.

A dynamic penalty function method was used to solve the optimization problems addressed in this article[16,17]. In this method the constrained optimization problem is transformed to the unconstrained minimization of a penalty function. A penalty is added to the cost function for any constraint violation. The unconstrained problem is solved by the single application of Snyman's trajectory method[16,17]. This method was chosen because it is a proven reliable and robust method which, through the dynamic adjustments of the penalty parameters, overcomes the problem of ill conditioning associated with traditional penalty function methods. In addition, graph theory was used to formulate the equations that describe the topology of the network.

The new design procedure was used to calculate the duct diameters for the network shown in *Figure 1*. The results of two case studies are discussed. In the first case study flow rates were specified in fifteen branches of the network. The duct diameters were then calculated so that the required flow rates were provided. In addition, the life cycle cost of the network was minimized. It will be shown that 20% savings on the life cycle cost of the network can be achieved by using the proposed design procedure. For the second case study, the same specifications were used, but velocity constraints were introduced to limit the noise generated by the ductwork. The algorithm produced an optimal design subjected to these contraints.

These two case studies were used to determine the effects of varying interest rates and energy costs on the capital and operating costs of optimized duct networks. The results indicate that for the network shown in *Figure 1* both the operating and capital cost of the optimum network are inversely proportional to the prevailing interest rate. As would be expected the operating cost is more sensitive to increases in the price of energy than is system initial cost.

The algorithm was also applied to the design of a pipeline for the transportation of coal. The coal has to be transported from a coal mine to a harbour port over a total distance of 768 km. Due to a lack of water at the coat mine a second pipeline is needed to transport water back to the mine. The total project cost is approximately $200 million. It will be shown that 14% ($8 million) can be saved on the pipeline by utilizing the proposed optimization procedure when compared with the previous design obtained by others with traditional methods.

A practical optimization procedure has therefore been developed to design pipe and duct networks. The procedure is currently being implemented in a user-friendly computer program which will assist engineers in effectively designing optimum networks.

FORMULATION OF THE OPTIMIZATION PROBLEM

The design can be rewritten as a constrained nonlinear optimization problem. The objective of the optimization is to determine the pipe or duct diameters that minimize the life cycle cost of the network. The life cycle cost consists of the capital and operating cost. The capital cost is calculated using thė 'weight-and-labour' method[18]. The mass of material in each duct section is multipled with a cost factor. This cost factor is the installed cost per kilogram weight of material used in the duct section. It accounts for the material, manufacturing and installation cost of that specific duct section. It can vary for different duct sections. For example, it may be more expensive to install a vertical duct than a horizontal one. The capital cost of a duct network is given by:

$$E_c = \sum_{i=1}^{n} C_i M_i \quad (1)$$

The material mass of each duct section can be expressed as a function of the diameter since the length and thicknesses are assumed constant.

The operating cost consists mainly of the cost of energy required by the fan. In present value terms the operating cost can be expressed as:

$$E_o = (PWEF)(E_d + E_e T) Q_F P_F \tag{2}$$

The present worth escalation factor is calculated by the following formula[11]:

$$(PWEF) = \frac{(1+I)^D - 1}{I(1+I)^D} + \frac{\delta E_e}{I}\left[\frac{(1+I)^D}{I(1+I)^D} - \frac{D}{(1+I)^D}\right] \tag{3}$$

In this equation it is assumed that the price of energy will increase linearly with time. Due to the uncertainty involved in estimating the rate with which the energy prices will escalate, a constant value is normally used over the entire system lifetime. In such a case the second term in (3) can be ignored.

The objective function is therefore given as:

$$E_L = E_C + E_O \tag{4}$$

The design specifications are introduced as constraints in the optimization problem. Typical constraints are specified flow rates, maximum allowable pipe diameter, minimum pressure to prevent cavitation and maximum allowable velocity to prevent erosion. Equality, as well as inequality contraints are therefore encountered.

Inclusive of the equality constraints are Kirchoff's laws, namely,

- node law: the total flow into any node of the network must be equal to zero,
- loop law: the sum of the pressure gains and losses in all the loop of the network must be equal to zero, and
- the element law: the flow through each network element must be related to the pressure loss over the element[19].

The first two laws arise from the topology of the pipe network and the last law from the physical properties of each network element. To develop a general purpose design program, a method has to be established which formulates the first two laws for a computer program. One method is to generate the governing equations for each network element and to balance the flow at each node. An equation is therefore obtained for every element and node in the network. This results in a sparse system of equations, some of which may be linearly dependent and thus difficult to solve.

Alternatively, graph theory may be used. It enables the formulation of the linearly independent Kirchoff equations that completely describe the topoplgy of the network. These equations are then solved during the optimization. This increases the efficiency and reliability of the solution algorithm[20].

PIPE AND DUCT NETWORK SIMULATION

Computer formulation of Kirchoff's laws

The first step in the solution procedure is to formulate the equations for Kirchoff's first two laws as defined in the previous section. To facilitate the computer solution of the problem two matrices have to be generated, i.e. the fundamental loop matrix and the cutset matrix. Graph theory is applied to the step-by-step process of intermediate matrix operations through which the topological information is transcribed to comply to Kirchoff's laws.

To apply graph theory to the formulation of the topology equations, a directed graph consisting of branches connected to the nodes in *Figure 2* must first be generated. Each branch represents a network element such as a pipe section or pump associated with its specified flow direction. This direction corresponds to the assumed direction of positive flow.

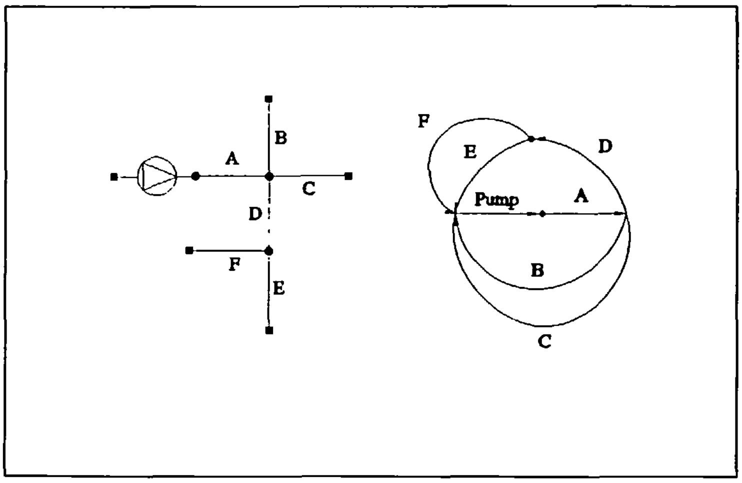

Figure 2 A pipe network and its directed graph

The topological information contained in this directed graph can be transcribed to a matrix called the incidence matrix[20]. The rows and columns of the incidence matrix correspond, respectively, to the nodes and branches in the directed graph. The elements of the incidence matrix are defined as:

- $a_{ij}=1$ if branch j is connected to node i and the arrow is pointing away from node i,
- $a_{ij}=-1$ if branch j is connected to node i and the arrow is pointing towards node i, and
- $a_{ij}=0$ if branch j is not connected to node i.

The incidence matrix for the directed graph in *Figure 2* is:

$$A=\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 & 0 \end{bmatrix} \quad (5)$$

The rows of this incidence matrix are not linearly independent. To obtain linear independence any row of the incidence matrix is omitted[20]. No information on the topology will be lost. The resultant matrix is called the reduced incidence matrix. The reduced incidence matrix for the pipeline example is obtained by excluding the final row:

$$A_R=\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (6)$$

The next step is to identify a 'tree' in the directed graph above. A tree is defined as a subgraph which contains all the nodes of the directed graph[20]. Furthermore, paths (which could be a combination of branches) must exist between each node in the subgraph. However, the subgraph may not contain loops. It can be shown that the tree consists of the branches in the directed graph corresponding to the linearly independent columns of the reduced incidence matrix. Standard numerical procedures exist to determine which columns of the reduced incidence matrix are linearly independent. One method is to write the matrix in its echelon form[20]. The linearly independent columns are incorporated into the tree matrix. By using these standard techniques this example's tree matrix is given by:

$$A_T=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{7}$$

Figure 2 shows that the subgraph with branches (Pump,a.d) contain all the nodes in the directed graph. Furthermore, paths exist between each node. However, the subgraph does not contain any loops. So, they form a tree! The columns in the reduced incidence matrix that are not in the tree are incorporated into the link matrix.

$$A_L=\begin{bmatrix} -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{8}$$

From the tree matrix, the reduced incidence matrix and the link matrix, two topology matrices are generated, i.e. the fundamental cutset and loop matrices.

$$CUTSET^*\ D_C=A_T^{-1}\cdot[A_T\vdots A_L]=\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 & 0 \end{bmatrix} \tag{9}$$

$$LOOP:\ B=[A_T^{-1}\cdot A_L\vdots 1]=\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

These matrices contain the minimum necessary information to describe the topology of the network and are used in the program to solve the problem. The equations for Kirchoff's laws are obtained by multiplying,

(a) the loop matrix with a vector of the pressure differentials for the loop laws, and
(b) the cutset matrix with a vector of the flows for the node laws.

The matrix operations, necessary for the formulations of the fundamental cutset and loop matrices, can easily be programmed. The topology of the network can therefore effectively be formulated on a computer.

Formulation of the element law

Now that Kirchoff's first two laws for nodes and loops have been defined, let us look at the third law, i.e. the element law. Two sets of equations have been obtained in terms of the flow and pressure differential for each network element. The pressure differential over each element can be related to the flow by the governing equations for that specific element. In the example, the coupling between the pressure differentials and flows for the pipes are given by the well-known Darcy Weisbach equations[2] as:

$$\delta P_i=f_i\rho\frac{L_i}{d_i}\frac{V_i^2}{2} \tag{11}$$

The friction factor f_i is calculated by Colebrook's equation.

Solving the topology equations

A new system of equations containing only the flows through each network element has now been obtained. This system of nonlinear equations may be solved in different ways, either by using standard methods such as Newton's or Broyden's method or by introducing the equations as equality constraints in an optimization program and solving them through an optimization procedure[21]. Newton's method has been used to solve the topolgy equations for the case studies presented in this article. The topology equations are solved each time the objective function is evaluated.

Optimizing the network

The constrained optimization problem can be generalized as:

$$\textit{minimize}\ f(X), \qquad X \varepsilon R^n \tag{12}$$

so that:

$$g_i(X)=0, \qquad i=1,2,\ldots,m \tag{13}$$

and:

$$h_j(X)\leqslant 0, \qquad j=1,\ldots,p \tag{14}$$

A dynamic penalty function method is used to solve the constrained optimization problem. Penalty function methods are the most basic and straightforward approach to constrained optimization problems[22,23]. The constraints are added as penalties to the original objective function resulting in an unconstrained optimization problem. The unconstrained optimization problem is given by:

$$\textit{Minimize}\ L(X)=f(X)+\sum_{i=1}^{m} P_i g_i^2(x)+\sum_{j=1}^{n} Q_j h_j^2(X) \tag{15}$$

where:

$$Q_j = \begin{matrix} 0 & \textit{if} & h_j(X)\leqslant 0 \\ \alpha_j & \textit{if} & h_j(X)>0 \end{matrix} \tag{16}$$

The choice of magnitude of the penalty parameters P_i and Q_j is critical. If too large penalty parameters are chosen the problem becomes ill-conditioned and difficulties are experienced with convergence. On the other hand, if the values are chosen too small the constraints are not accurately satisfied because the effect of $g_i(X)$ and $h_j(X)$ on the modified objective function is reduced.

A possible solution to this problem is to use variable penalty parameter values. Initially, small penalty parameter values are chosen and the optimization is performed to give a corresponding solution. The penalty parameter values are then systematically and sequentially increased. The solution to the previous problem is used as the starting values for the current problem. This procedure is repeated until the increase in penalty parameter values no longer affects the solution. This procedure is the well-known Sequential Unconstrained Minimization Technique—SUMT[22]. The disadvantage of this method is that it requires the repetitive solution of the unconstrained optimization problem. To overcome this problem, we apply here a novel dynamic penalty parameter procedure proposed by Snyman[16,17] in which he systematically increases the penalty parameters during a single application of his dynamic trajectory method[16,17] for unconstrained minimization.

The basic method involves the solution of the equations of motion for a particle of unit mass in a conservative force field. The potential energy of the particle represents the objective function to be minimized. The algorithm monitors the kinetic energy of the particle and via an interfering

Table 1 Input data for case study 1

Design specifications: Required flow rates in duct sections (*E,F,G,L,M,N,S,T,U,Z,AA,AB,AG,AH,AI*) are 1.0 kg/s		
Fluid (Air)	Density	1.018 kg/m³
	Viscosity	0.000017 Ns/m²
Economic data	Interest rates (II)	15%
	Design life time (D)	20 years
	Electricity rate (E_e)	8.62 c/kWh
	Electricity demand rate (E_d)	R28/kW/month
	Energy rate increase (δE_g)	0 c/kWh/year
	Operating hours (T)	6570 hours/year
	Pipe material cost factor (C)	R13/kg
Duct data	Duct material	Galvanized iron
	Surface roughness (for use in Colebrook's equation to determine friction factor)	0.05
	Duct thickness	10 mm
	Duct thickness material density	7000 kg/m³
	Length	
	Sections *A,H,O,AC*	4 m
	Sections *B,I,P,W,AD,D,K,R,Y,AF*	7 m.
	Sections *E,F,G,L,M,N,S,T,U,A,AA,*	5 m.
	AB,AG,AH,AI	5 m.
Restrictions	Maximum velocity	*Case study* 1—none
		Case study 2—10 m/s

strategy ensures that the total energy of the particle is systematically reduced, resulting in a trajectory to a point of minimum potential energy. The objective function is therefore minimized. At each step along the trajectory of the particle the gradient of the objective function is calculated. This gradient is used to calculate the next step of the particle. The basic minimization method, requiring only gradient information and no explicit line searches, has been found to be extremely robust and reliable[16,17]

In applying this method to the constrained problem, the penalty function parameters are systematically increased each time the gradient of the objective function is evaluated along the computed trajectory. This is done until certain prescribed penalty parameter values are reached after which the penalty parameters are kept constant. The trajectory is then continued with the constant penalty parameter values until convergence is achieved. In this study the initial values used for the penalties are $P_i^0 = Q_j^0 = 100$. These values are stepped up by a factor 1.9 at each step along the trajectory until the maximum values of $P_i^{max} = Q_j^{max} = 100\,000.0$ are reached. Thereafter, in the further computation of the solution path, the parameters are kept constant at the maximum values.

Due to the complexity of the optimization problem the modified objective function will contain a large number of local minima. The only way to determine when the global minimum has been reached is ideally to locate all the local minima and then choose the best one of these. A study was conducted to determine if the algorithm converges to the same solution irrespective of the initial guesses. If this is the case it is likely that the solution achieved is the global minimum.

For both the case studies discussed below the algorithm converged to the same solution.

APPLICATIONS

Two case studies were performed on the duct network shown in *Figure 1*. The input for the case studies is shown in *Table 1*. These case studies and the results obtained will now be discussed.

Table 2 Results of case studies 1 and 2

Case study 1				Case study 2			
Operating cost (R)	39283			Operating cost [R]	39062		
Capital cost [R]	8155			Capital cost [R]	9858		
Section	D (m)	Section	D (m)	Section	D (n)	Section	D (m)
A	1.4717	*U*	0.4210	*A*	1.3820	*U*	0.4252
B	0.6302	*V*	0.9602	*B*	0.6180	*V*	0.8740
C	0.4092	*W*	0.6685	*C*	0.5046	*W*	0.6180
D	0.4041	*X*	0.5689	*D*	0.4250	*X*	0.6028
E	0.3213	*Y*	0.4296	*E*	0.3568	*Y*	0.4553
F	0.3393	*Z*	0.3367	*F*	0.3566	*Z*	0.3568
G	0.4043	*AA*	0.3607	*G*	0.4252	*AA*	0.3822
H	1.3292	*AB*	0.4298	*H*	1.2361	*AB*	0.4554
I	0.6443	*AC*	0.6891	*I*	0.6180	*AC*	0.6180
J	0.5483	*AD*	0.6894	*J*	0.5046	*AD*	0.6180
K	0.4141	*AE*	0.5867	*K*	0.4250	*AE*	0.6028
L	0.3246	*AF*	0.4431	*L*	0.3568	*AF*	0.4553
M	0.3476	*AG*	0.3473	*M*	0.3568	*AG*	0.3568
N	0.4142	*AH*	0.3720	*N*	0.4252	*AH*	0.3822
O	1.1624	*AI*	0.3473	*O*	1.0705	*AI*	0.4554
P	0.6548			*P*	0.6180		
Q	0.5573			*Q*	0.5040		
R	0.4209			*R*	0.4250		
S	0.3299			*S*	0.3568		
T	0.3533			*T*	0.3568		

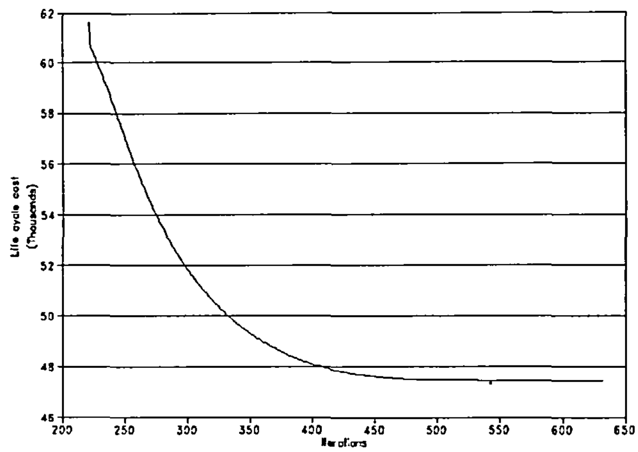

Figure 3 Convergence of algorithm

Case study 1

In the first case study flow rates of 1 m^3/s were specified in branches (*E,F,G,L,M,N,S,T,U,Z, AA,AB,AG,AH,AI*) of the network. The diameters of the duct sections were calculated so that the required flow rates were delivered at these specific branches. In addition the life cycle cost was minimized. The results are given in *Table 2*. *Figure 3* depicts the decrease in cost for subsequent

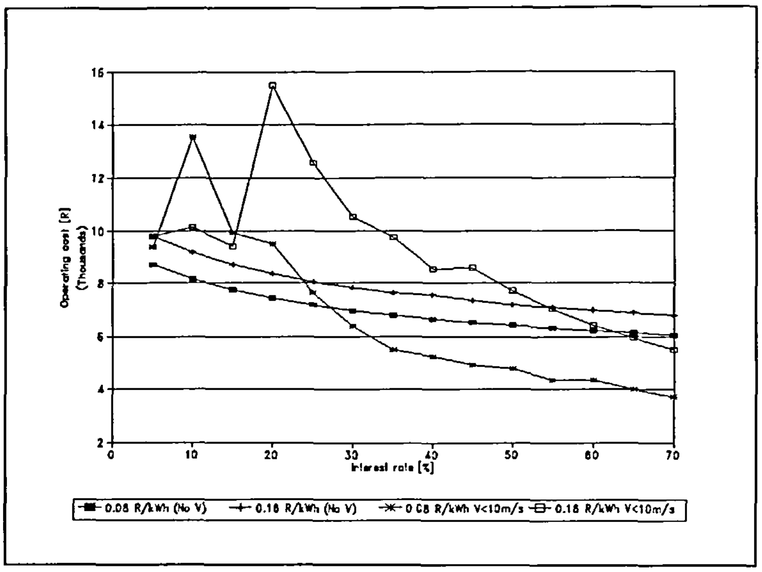

Figure 4 Operating cost with varying interest rates and energy cost

iterations. For each of these iterations in *Figure 3* the constraints are satisfied within an accuracy of 4% without requiring dampers installed in the flow. All these iterations on the graph represent workable network solutions. When traditional non-optimization methods are used for duct design it is possible that any point on this curve could be the solution. A study conducted on different networks indicated that up to an average of 20% could be saved when the proposed optimization procedure is used.

Case study 2

In this case study the velocities in the duct sections were limited to a maximum value of 10 m/s. Typically, these constraints are added to prevent noise generation. The same optimization as in case study 1 was then performed but with the addition of the inequality constraints. The results in *Table 2* show that the addition of constraints results in an increase in the life cycle cost of the network. Furthermore dampers have to be installed in all the paths of the network except for the longest path i.e. path (*A,H,O,V,AC,AD,AD,AF,AI*). This is necessary because the flow in the shorter paths cannot be restricted by decreasing the duct diameters which will result in higher duct velocities and more noise generation.

Effects of varying interest rates and energy prices

Calculations similar to those discussed in *case study 1* and *2* were performed to illustrate the effects of varying interest rates and energy cost on the capital, operating and life cycle cost of the optimized network in *Figure 1*.

Figure 4 depicts the operating cost of the optimized network for interest rates varying from 5 to 70% and energy costs of 0.08 R/kWh and 0.16 R/kWh, respectively. The variations in capital and life cycle costs are represented in *Figures 5* and *6*, respectively. The curves denoted by the plus sign and filled rectangle were calculated without velocity restrictions as in *case study 1*. The other two curves on these three graphs were calculated with velocity restrictions of 10 m/s in all the branches of the network as in *case study 2*.

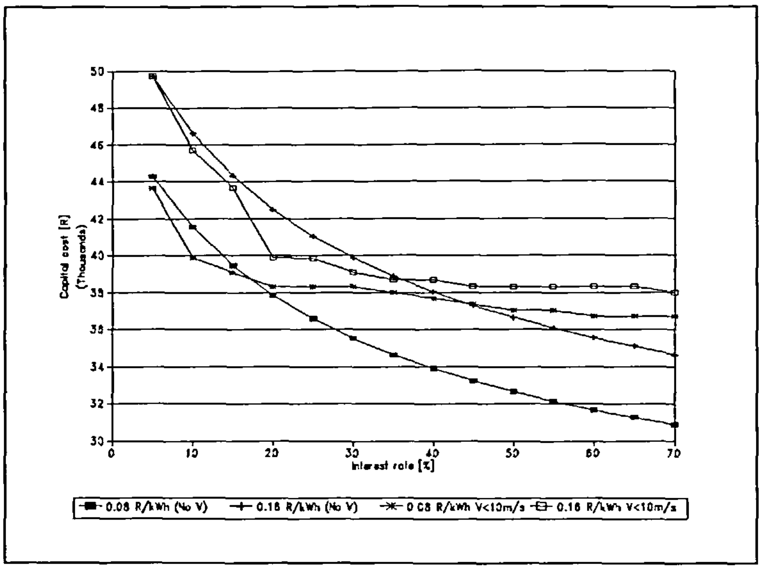

Figure 5 Capital cost with varying interest rates and energy cost

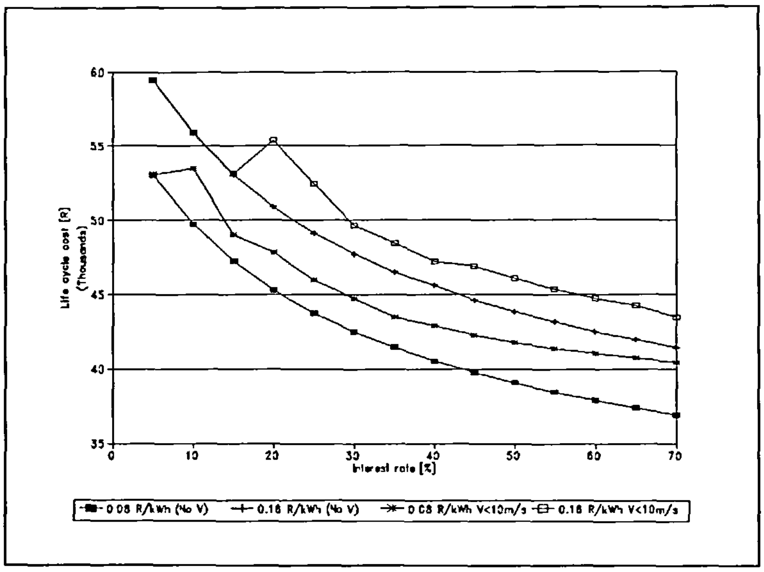

Figure 6 Life cycle cost with varying interest rates and energy cost

From *Figures 4* and *5* it can be seen that both the operating and capital costs of the optimized network decrease as the interest rate increases when no restrictions are placed on the velocities. Furthermore, the operating cost and capital cost increase when the energy cost is doubled. Consequently, the life cycle cost of the optimized network decreases as the interest rate increases. This is due to the fact that the present value of any future investment decreases considerably as the interest rate increases. The life cycle cost increases when the energy cost is doubled as depicted in *Figure 5*.

When velocity restrictions are introduced the curves are characterized by a step-like behaviour. From *Figure 5* it can be seen that the capital cost of the optimized network remains more or less constant with interest rates larger than 20%. This is due to the fact that the duct diameters have reached their minimum possible values. Any further reduction the duct dimensions would result in velocities exceeding 10 m/s. The dimensions of the distribution network is therefore fixed. The operating cost decreases as the interests rate increases for interest rates larger than 20%.

For interest rates lower than 20% the operating cost increases as shown in *Figure 4*. This increase is balanced by a more rapid decrease the capital cost of the network shown in *Figure 5*. The result is an increase the life cycle cost of the network as depicted in *Figure 6*. The steps in the curves in *Figure 6* occur when the velocity restrictions begin to play a role and the capital cost of the network stabilizes i.e. at interest rates of 10 to 20%.

From these studies it can be concluded that the operating cost and capital cost of the optimized network are inversely proportional to the prevailing interest rates. Furthermore the capital cost of the network decreases as long as the velocities are permitted to increase in each section until minimum dimensions are attained. From this condition onwards the capital cost remains more or less constant and only the operating cost can decrease.

Case study—Coal pipeline

This case study was chosen because a detailed design had already been performed by outside consultants with traditional design methods. A meaningful comparison could therefore be made. Both the pipelines used in the transportation of coal have four pumping stations. The sites of these stations were determined after a detailed study of the environment had been made. The diameters of the pipe sections linking the pumping stations were optimized. As the pipes will be manufactured on site and therefore do not require standard diameters, a continuous optimization algorithm could be applied.

The objective for the optimization was to minimize the life cycle cost of the network. The life cycle cost was calculated in much the same way as the previous case studies. The results indicate that a total saving of 14% on the total project cost of $200 million can be achieved when the pipe diameters are optimized. This corresponds to a 20% saving on the pipeline itself.

CONCLUSIONS

An effective procedure for the optimum design of flow networks has been developed. The procedure is implemented in a flexible and user-friendly computer program. The program uses graph theory and a dynamic penalty function method to optimize the design of pipe networks. It has been successfully applied to the design of duct and pipe networks of different sizes and complexity. Although the program is already a practical design tool, further potential exists for improvement and extension of its capabilities. Typical future work will include the expansion of flow elements to include multiple inlet and outlet ports.

The ultimate objective is to produce user-friendly computer programs for application in specialized engineering disciplines such as waste water reticulation, air conditioning and pipeline engineering. The justification lies in the fact that networks form a large percentage of certain projects, i.e. 15% to 20% of the capital cost of a chemical installation. This percentage is substantially higher for oil and coal pipelines. When one considers that the cost of a coal pipeline is in the order of several hundred million dollars a percentage point saving is a considerable amount, showing the value of future research in optimal design of pipe networks.

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